Emergence of dominant opinions in presence of rigid individuals

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Abstract In this chapter, we study the dynamics of the so-called naming game as an opinion formation model with a focus on how the presence of a set of rigid minorities can result in the emergence of a dominant opinion in the system. These rigid minorities are "speaker-only", i.e., they only "speak" and never "listen" thus strongly affecting the course of a social agreement process. We show that for a moderate α (fraction of rigid minorities), the agreement dynamics results in an emergence of a dominant opinion. We extensively study the property of such dominant opinions and observe that the dominance is not the characteristic property of only the "speaker-only" opinions; other opinions under certain circumstances can also become dominant. However, with increasing α, the chances of a "speaker-only" opinion becoming dominant increases. We also find early invented opinions possess higher chances of becoming dominant. We embed this model on various static interaction topologies and real-world time-varying face-to-face interaction data. Importantly, for a reasonably static societal structure the presence of rigid minorities influences the emergence of a dominant opinion to a much larger extent than in case where the societal structure is very dynamic.

1 Introduction

Our social behavior is to a large extent determined by the society we live in. The social interactions among different individuals in a society shape/reshape or may determine how an individual will adopt new ideas or opinions. A group of individu-

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als that strongly advocate compelling points of view can influence public sentiments and opinions on a particular issue. This phenomenon is evident in multi-party election campaigns where politicians belonging to multiple parties try to influence the public and try to align their opinion toward that of the politician's party. Each party here represents a political ideology and is therefore representation of a rigid opinion. Each politician bearing the rigid political ideology of a particular party tries to win the votes of other individuals in the population. Each group of the rigid individuals convey the opinion of the party. Thus, the rigid group of individuals who never change their opinions play a vital role in opinion formation and may influence the agreement process. Such type of competition is also valid for describing the language competition among variety of languages where a very few survive while a majority die competing with others [1, 2, 3, 4].

In this chapter, we focus on the popular naming game framework (NG) [5] as a model of opinion formation to study how social dominance emerge in the presence of rigid individuals and how such dominance could influence societies to align toward the dominant opinion through negotiation. The naming game is a simple multi-agent model that employs local communications which leads to the emergence of a shared communication scheme/common opinion in a population of agents. The system evolves through pairwise interactions among agents that necessarily capture the generic and essential features of an agreement process. This model was conceived to explore the role of self-organization in the evolution of languages [6, 7]. Steels in his paper [6] primarily focused on the formation of vocabularies, i.e., a set of mappings between words and their meanings (for physical objects). In this context, each agent develops its own vocabulary in a random and private fashion. Agents are forced to align their vocabularies, through successive conversation, in order to obtain the benefit of cooperating through communication. Thus, a globally shared vocabulary emerges as a result of local adjustments of individual word-meaning associations. The communication evolves through successive conversations. It is worth pointing out that these conversations are particular cases of language games, which are used to describe linguistic behavior but can also describe non-linguistic behavior, such as pointing. As a practical example of this model, Talking Heads experiment in [8] was carried out where robotic agents develop their vocabulary observing objects through digital cameras, assigning them randomly chosen names and sharing these names in pairwise interactions. This model has also acquired attention in the field of semiotic dynamics [9, 10] that studies evolution of languages through invention of new words, grammatical constructions and more specifically, through adoption of new meanings for different words. For instance, the proliferation of new generation of web-tools enabling human web-users to self-organize a system of tags in such a way to ensure a shared classification of information about different arguments, for example, del.icio.us or www.flickr.com has took place [10].

The minimal naming game (NG) consists of a population of N agents observing a single object in the environment (may be a discussion on a particular topic) and opining for that through pairwise interactions, in order to reach a global agreement. We consider discrete opinions and the all the opinions to be distinct. We also assume the opinions to be uncorrelated. The agents have at their disposal an internal inventory, in which they can store an unlimited number of different words or opinions. At the beginning, all the individuals have empty inventories. At each time step, two individuals are chosen - the “speaker” chosen randomly and the “hearer” also chosen randomly but from the neighborhood of the “speaker”. The speaker voices to the hearer a possible opinion for the object under consideration; if the speaker does not have one, i.e., his inventory is empty, he invents a brand new opinion which is completely different from other opinions present before it. In case where he already has many opinions stored in his inventory, he chooses one of them randomly. The hearer’s move is deterministic: if she possesses the opinion pronounced by the speaker, the interaction is a “success”, and both speaker and hearer retain that opinion as the right one, removing all other competing opinions/words from their inventories; otherwise, the new opinion is included in the inventory of the hearer, without any cancellation of opinions in which case the interaction is termed as a “failure” (see fig 1).

We recast this model to incorporate a small set of rigid individuals and investigate the effect of their presence on the overall dynamical properties of the system. One of them could be the unwillingness to listen. This type of rigid individuals try to speak a lot and never listen to the others. Therefore, these rigid individuals take part in the interaction only as “speaker”, and never as hearer. Since, an adoption of an opinion is only possible in the role of a hearer, these rigid agents never undergo any change in their opinion. The interactions between pairs of rigid individuals are therefore forbidden as they can never influence one another. Consequently, these rigid/“speaker-only” agents do not allow the population to reach a global consensus except the case where there is only one “speaker-only” agent in the population and rest of the population adopts the opinion of this particular agent in order to reach final agreement. However, in general, stable polarized/multi-opinion states are observed in the system at long times; such multi-opinion states have also been reported by [11] however, in a different context of modeling trust among agents. Examples of such forms of rigidity are found in languages quite frequently; for instance, some languages are very rigid in their word order (e.g., Irish, English, Persian) as opposed to certain others (e.g., Turkish, Russian) that are very flexible [12].

The rest of the chapter is organized as follows. In section 2, we discuss the state-of-the-art. In section 3, we describe the model in detail. Section 4 is devoted for the discussion and analysis of interesting insights that we obtain from the model in presence of a set of “speaker-only” agents for different social structures. In section 4, we draw conclusions and point to future direction of this research.

2 Related Work

Opinion dynamics models involving committed individuals have been studied previously in [13, 14, 15, 16, 17, 18, 19, 20, 21]. [13] studied how the presence of zealots (rigid individuals) affected the distribution of opinions in the case of the voter model. Similarly, [13], [18] studied the properties of steady-state opinion

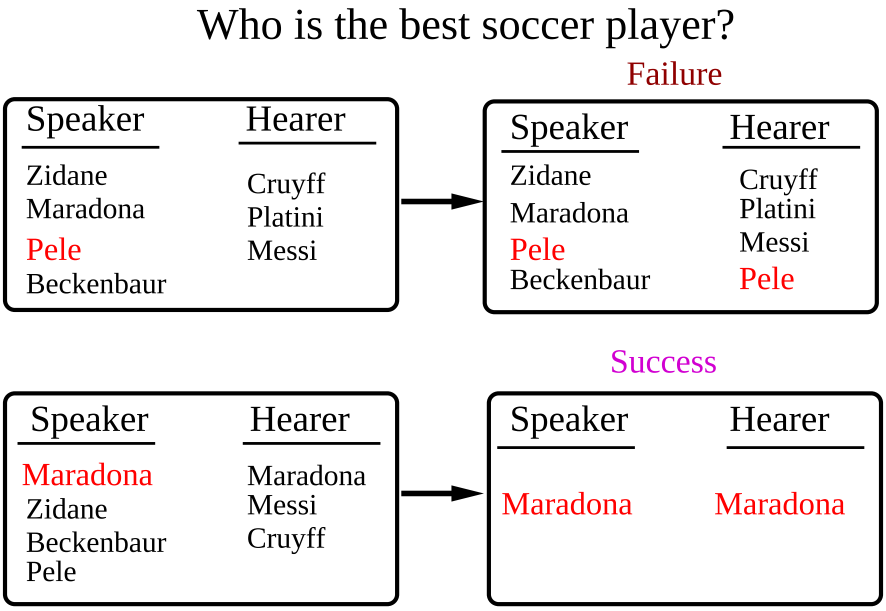

Fig. 1 Agent's interaction rules in NG on a topic, say "Who is the best soccer player?". The speaker selects the opinion highlighted. If the hearer does not possess that opinion she includes it in her inventory and the interaction is a "failure" (top). Otherwise both agents erase their inventories only keeping the winning opinion and interaction is a "success" (bottom). .

distribution for the voter model with stubborn agents, but additionally considered the optimal placement of stubborn agents so as to maximally affect the steady-state opinion on the network. The effect of rigid/committed individuals has also been studied on the binary naming game [19, 20, 21] where they introduce a set of individuals committed to a single opinion and study how majority opinion get rapidly reversed by the presence of a small fraction of them. In our model, we consider every "speaker-only" agent to possess one different opinion each and try to investigate the competition dynamics in the population finally, leading to the emergence of a single dominant opinion which is held by a majority of the agents. However, when the fraction of "speaker-only" agents crosses a critical value in the population, no single dominant opinion emerges, instead almost equal-sized multi-opinion clusters are formed. We further investigate the specific properties of the opinion that manifests as the most dominant one. In particular, we observe that those opinions that are invented early and, in most cases, by one of the "speaker-only" agents get elected as the dominant one. Nevertheless, for a reasonably low fraction of "speaker-only" agents an early invented opinion from the "non-speaker-only" group may also emerge as the most dominant one - we investigate in details the properties of these opinions in order to precisely reason for such a non-intuitive phenomena.

3 The model description

In this section, we discuss the model in detail. The population consists of N agents and out of them αN no. of agents are “speaker-only” agents. In each iteration of the game at $t = 1, 2, ...$, the following happens :

1. An agent is randomly selected from the population and act as “speaker”.
2. Another agent is chosen among the rest of the population except the “speaker-only” agents and is designated as “hearer”
3. If there is no name for the object/topic the “speaker” invents one brand new name otherwise he selects from list of already existing names/words and conveys it to the “hearer”. Please note that every agent has an inventory/memory to keep the names.
4. The hearer then searches for the topic name in her inventory. If the search is successful, then the game is called “successful game” and both the agents delete all other competing names from their inventory.
5. On an unsuccessful search, the “hearer” learns the name (adds the name/word into the already existing name-inventory) and the game is termed as “failure” interaction.

The emergent properties that are of interest in the naming game are the sum of memory sizes of all agents at a particular time step t denoted by $N_w(t)$, the number of unique words/opinions/names in the system at t ($N_d(t)$) and the time needed to reach the global consensus (t_{conv}).

4 Results and discussion

In this section, we shall try to elaborate the impact of having a fraction of “speaker-only” agents (α) in the population embedded on various types of social graphs and point to possible explanations of our findings.

4.1 The Mean-field case

The mean-field case corresponds to a fully connected network in which all agents are in mutual contact. Thus, every individual can, in principle, talk to every other individual. On this topology, we try to investigate the microscopic activity pattern of the game dynamics driven by the parameter α where α is the fraction of “speaker-only” agents chosen uniformly at random from the population. In figs 2(a) and (b), we have shown the time evolution of $N_w(t)$ and $N_d(t)$ where we observe that the system does not reach consensus and we have αN number of opinions left even after 5x10^7 games. The acquired state is the steady state which does not change as we have already reached the lower bound on the number of opinions with small fluctuations of N_w. Therefore, it is apparent that the system breaks down into clusters of opinions. Now, if we find the typical cluster sizes, we observe that there is usually one cluster which is extremely large compared to the other clusters. In other words, the distribution of the frequency of opinions (describing the number of agents who have possessed the opinion in their inventories) show skewness for lower values of α (see figs 2 (c) and (d) describing the relation between the frequency of opinions($freq_r$) vs the rank (r)) pointing to the emergence of a dominant opinion (the opinion that is present in most of the agent's inventories) in the system. However, for a large enough α, the frequency of winner decreases allowing an increase in the size of the other clusters. To find the cut-off α for which this phenomenon happens, we observe the dependence of the frequency of the dominant opinion (f_w) on the "speaker-only" fraction α and find a "mirrored" S-shaped curve - the first part shows a linear decrease, then an abrupt fall and then again a steady linear dependence (see figs 2(e) and (f)). The cut-off value of α decreases with increasing N.

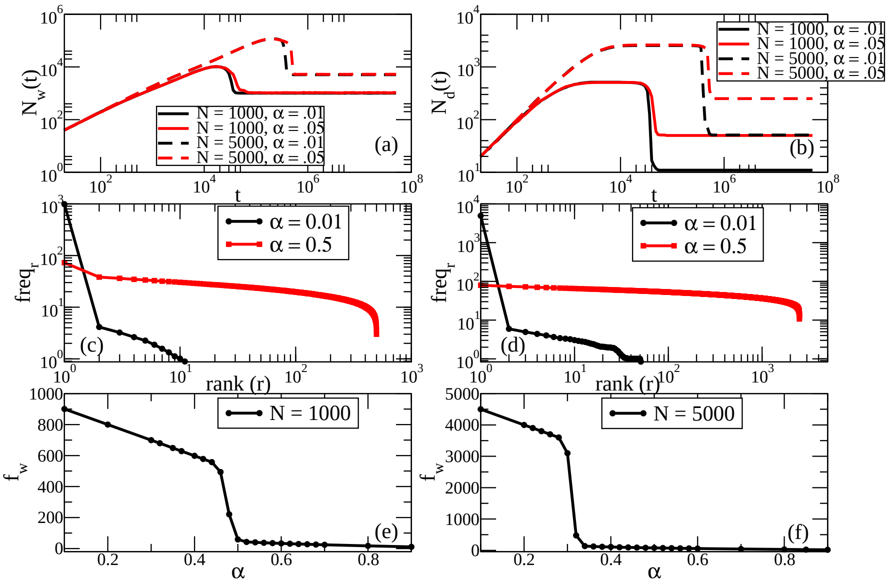

Fig. 2 Time evolution of (a) $N_w(t)$ and (b) $N_d(t)$ for N = 1000, 5000 and α = 0.01, 0.5. Frequency of the opinions ranked in order of decreasing frequency ($freq_r$) present in the system after 5x10^7 games for (c) N = 1000 (d) N = 5000. Relation between the frequency of the dominant opinion (f_w) and α for (e) N = 1000 (f) N = 5000. The curves are averaged over 100 simulation runs.

In particular, we shall attempt to understand the reasons for the high frequency of the dominant opinion. Therefore, the natural question that arises is : Why does a sheer majority in the population agree to this opinion? Do the "speaker-only" opinions (opinions invented by "speaker-only" agents) always emerge as winners? The answers to these questions can be found out from fig 3 where in fig 3(a), we show the winning probability of the “speaker-only” opinions (W_{sp}) and its variation with α. We refer to the winning probability as the fraction of simulations in which a “speaker-only” opinion is elected as the most dominant one. We observe that for a small enough α, the chance of winning of a “speaker-only” opinion is more than 50% and as we increase N the required value of α to guarantee more than 50% winning chance is even lowered. Therefore, for a moderate α, “the speaker-only” opinions suppress the chances of other opinions’ survival in the system and create a sheer monopoly for themselves.

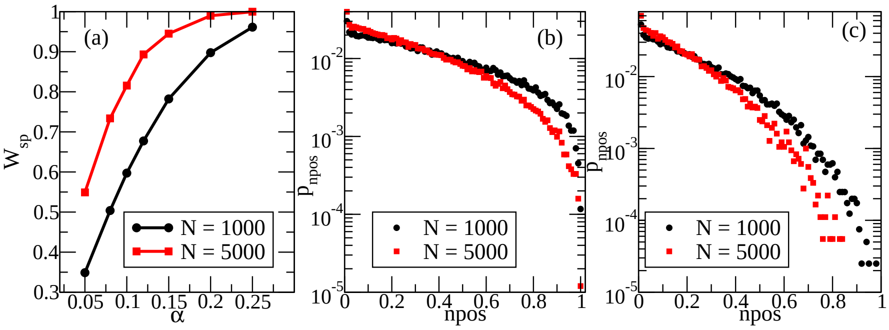

Fig. 3 (a) Relation between the winning probability of “speaker-only” opinions (W_{sp}) and α after 5x10^7 games for different N. Probability that a given opinion becomes the dominating one (p_{npos}) is plotted as a function of the normalized invention position ($npos$) when b) “speaker-only” opinions c) other opinions become the winner for α = 0.1. The curves are averaged over 10^5 simulation runs.

Furthermore, we shall observe the effect of the creation time of the opinions on the phenomenon of emergence of dominance in the system. It turns out that the early invented opinions are in advantageous position in this context irrespective of whether they are invented by a “speaker-only” agent or by any other agent in the system. In fig 3 (b) and (c), we plot the probability for an opinion to become the dominating one as a function of its normalized creation position. This means that each opinion is identified by its creation order: the first invented opinion is labeled as 1, the second as 2 and so on. To normalize the labels, they are then divided by the label of the last invented opinion. Clearly, the early invented opinions have higher chances of becoming dominant compared to the others invented late in time. To explore the property of the dominant opinion further, we delve deeper into the dynamics to identify the characteristic properties of the competing opinions invented before the dominant opinion. From table 1, it is clear that the dominant opinion has a fewer number of competitors facilitating its emergence as a winner. For an opinion to become winner, there has to be less number of competing opinions invented before it. In addition, from among “speaker-only” and “non-speaker-only” opinions, the proportion of “speaker-only” ones invented before the winner is significantly lower since they have a higher potential to quickly modify the opinion of the others. The presence of the “speaker-only” opinions are even much less in case a winner is a “non-speaker-only” opinion. The number of such opinions are approximately half as compared to the case when a “speaker-only” opinion emerges as winner. Now, if we observe the typical distribution of “speaker-only” opinions invented before the wining opinion, we find exponential-like distribution for both the cases (see figs 4(a) and (b)). Therefore, fewer the number of “speaker-only” opinions invented before an opinion, more is its chance to become the dominant or the wining opinion. This quantity is even lesser for the case when the dominant opinion has been invented from the “non-speaker-only” group as is also evident from the figure.

Table 1 Number of different types of opinions invented before the winner after 5x10^7 games, averaged over 10^5 simulation runs. “spo” refers to “speaker-only” opinions.

N	α	Winner	Earlier opinions (spo)	Earlier opinions (Rest)
1000	0.10	Spo	20.6	151.4
		Rest	12.1	96.4
	0.12	Spo	25.2	149.1
		Rest	14.3	92.7
5000	0.10	Spo	86.1	647.2
		Rest	47.3	385.8
	0.12	Spo	105.9	641.4
		Rest	55.9	369.7

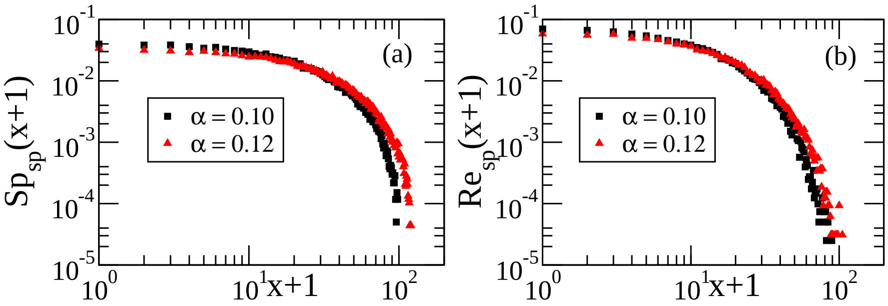

Fig. 4 Distribution of the number of “speaker-only” opinions before the winner when (a) “speaker-only” opinion ($Sp_{sp}(x)$) (b) “non-speaker-only” opinion ($Re_{sp}(x)$) is the winner for $N = 1000$. The curves are averaged over 10^5 simulation runs.

Although we observe that the presence of fewer competitors invented before the invention of a particular opinion increases its chance of becoming dominant, an important question is that why exactly one of them emerge as a winner? It seems that the winning opinion takes part in a significantly larger fraction of successful interactions compared to the other competing opinions created before it till the dominance time which helps in propelling its dominance in the system. We define dominance time as the time when the most frequent opinion in the system reaches frequency $\sim (1-\alpha)$N and next most frequent opinion has frequency less than 10% of the population. This phenomenon is supported by table 2. The competition seems to be even less pronounced if the winner has to be from the rest of the population as compared to the case where a "speaker-only" opinion is the winner.

Table 2 Percentages of successes for the winning opinion compared to other opinions invented before it till dominance time, averaged over 10^5 simulation runs.

N	α		Winner	Earlier opinions
1000	0.10	Spo	88.9	5.36
		Rest	93.1	3.07
	0.12	Spo	89.0	6.2
		Rest	93.5	3.42
5000	0.10	Spo	91.0	5.8
		Rest	94.34	3.38
	0.12	Spo	90.67	6.64
		Rest	94.77	3.46

4.2 Scale-free networks

Social networks are far from being fully-connected or homogeneous. Most of them show a large skew in the distribution of node-degrees resulting in the so-called scale-free networks. In this section, we shall study the effect of α on the Barabási & Albert (BA) network [22] which follows a scale-free degree distribution. Since low degree nodes form a vast majority in such networks, any randomly chosen node is, with high probability, a low degree node. The neighbors of this low degree node, however, should be high degree nodes (since the high degree nodes tend to be connected to almost all low-degree nodes by virtue of their high degree). Therefore, in this case we adopt two strategies while selecting the α fraction of "speaker-only" agents - (a) we select uniformly at random and (b) we select preferentially based on degree so that the high-degree hubs are more probable of being selected as a "speaker-only" participant.

Similar to the mean-field case, we observe a single dominant opinion in the system with majority of agents aligning to this opinion. Nevertheless for higher α, this dominance of a single opinion does not persist in the system (see figs 5 (a), (b), (e) and (f)). The decomposition of the system into multiple similar size clusters occurs for a lesser value of α as compared to the mean-field case. In fact, this decomposition is even more pronounced for the case where the "speaker-only" population is chosen preferentially. A similar mirrored S-shaped dependence of the frequency of the winning opinion on α is again observed. However, the sharp transition occurs at a much lower α here and more so in case of preferential selection of the "speaker-only" agents (see figs 5 (c), (d), (g) and (h)). This is the consequence of the fact that social networks are sparse (with agents mostly interested in the local neighborhood) and therefore more vulnerable to such decomposition than the mean-field scenario. Further, selecting preferentially the hubs as the "speaker-only" agents help mani-

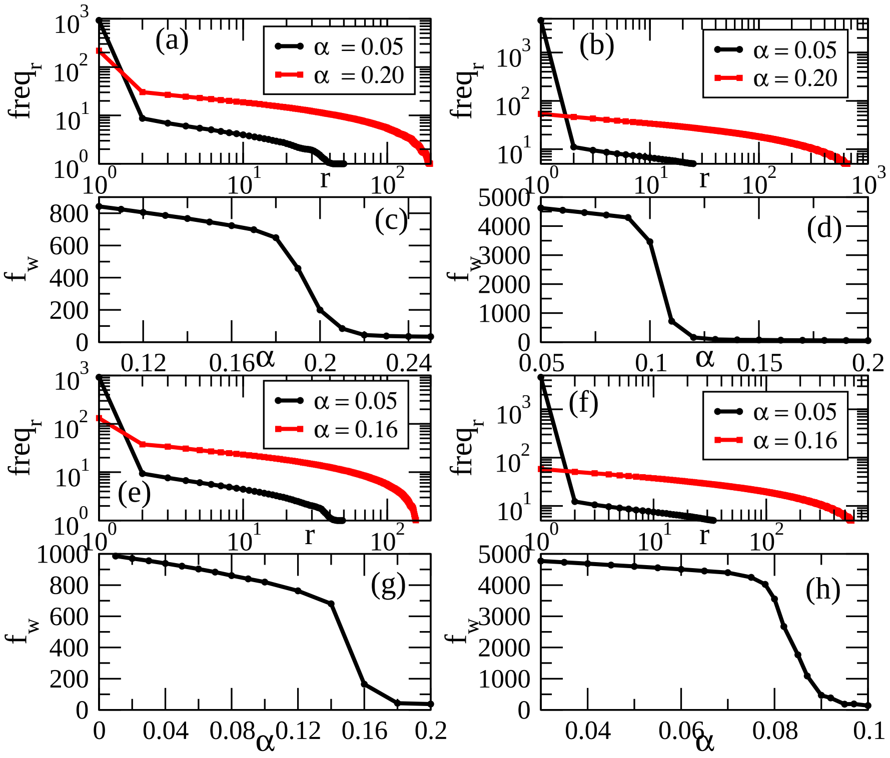

Fig. 5 $freq_r$ vs r after 5x10^7 games for "speaker-only" agents distributed randomly with (a) N = 1000, (b) N = 5000 and preferentially with (e) N = 1000, (f) N = 5000 respectively. f_w vs α for (c) N = 1000, (d) N = 5000 when "speaker-only" agents are selected randomly and (g) N = 1000, (h) N = 5000 when "speaker-only" agents are selected preferentially. The curves are averaged over 100 simulation runs for 10 network realizations each.

fold in propelling their opinions very fast in the population mostly because of their disproportionately high connectivity. Next we study the relationship between the winning probability of the "speaker-only" opinions and α. The winning probability of the "speaker-only" opinion increases as we increase α. The typical α for which the winning probability becomes 50% is much lower compared to the mean-field case (see figs 6 (a) and (b)).

The creation time of an opinion plays an important role in deciding the dominance of the opinion. For the case when a "speaker-only" opinion becomes winner, irrespective of the way the "speaker-only" agents are selected (random/preferential) the lately invented opinions seem to mostly emerge as the winner which is possibly due to the inherent skewed degree distribution of agents (see fig 6 (c), (d)). Note that this result is markedly in contrast with those observed for the mean-field case. It is actually the late inventors in the "speaker-only" population who seem to be in advantageous position because by being late they are able to align their local neighborhood to their opinion in the last stages (i.e, there are no further scopes of opinion

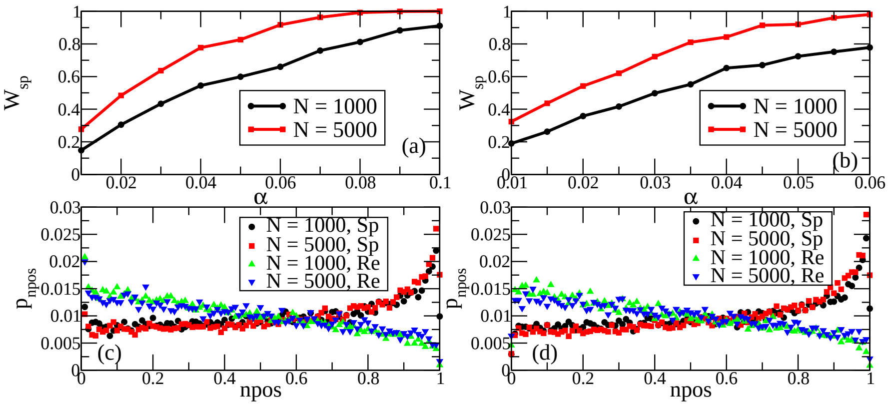

Fig. 6 W_{sp} vs α after 5x10^7 games when the "speaker-only" agents are selected (a) randomly (b) preferentially from the population. p_{npos} vs $npos$ when "speaker-only" opinions (Sp) and other opinions (Re) become the winner with "speaker-only" agents (c) randomly and (d) preferentially selected respectively for $\alpha = 0.03$. The curves are averaged over 10^4 simulation runs for 10 network realizations each.

switch) and the larger this neighborhood, the higher is the chance that the lately invented "speaker-only" opinion is the winner. Further, a "speaker-only" agent has mostly "non-speaker-only" agents in his local neighborhood which makes it easy for the "speaker-only" agent to align them to his opinion and the later this takes place, the higher is the chance that this "speaker-only" opinion would emerge as the winner. However, the early invented opinions seem to be more probable winners in case this opinion has been initiated by a "non-speaker-only" member of the population (see fig 6 (c), (d)). We observe that the number of "speaker-only" opinions invented before the dominant opinion is always less in case this opinion is from the "non-speaker-only" group as in the mean-field scenario (see table 3). In addition, the number of "non-speaker-only" opinions invented before the winner is larger compared to the "speaker-only" opinion pool. We also analyze the proportion of successes of the winner in comparison to the opinions created before the invention of the winner till the dominance time and observe that a huge majority of the successful interactions are witnessed by the winner (see table 4). Although the mean-field trends are preserved here, one important point of difference is that the proportion of successes witnessed by the competitors of the winner is much larger here indicating a tougher competition.

5 Time-varying networks

In all the above discussions, we have considered static graphs where the link structure is known a priori and the game is played on top of the same. However, real-world social networks show dynamicity. Links appear and disappear over time.

Table 3 Number of different types of opinions invented before the winner, averaged over 10^4 simulation runs for 10 network realizations each when the “speaker-only” agents are randomly selected. The numbers in parentheses correspond to the case of preferential selection of “speaker-only” agents.

N	α	Winner	Earlier opinions (spo)	Earlier opinions (Rest)
1000	0.01	Spo	3.9 (3.9)	318.9 (320.8)
		Rest	3.1 (3.1)	240.1 (240.5)
	0.03	Spo	12.8 (13.0)	315.5 (316.6)
		Rest	9.2 (9.2)	234.6 (234.2)
5000	0.01	Spo	22.6 (22.4)	1670.3 (1654.7)
		Rest	16.0 (16.0)	1245.3 (1247.0)
	0.03	Spo	69.4 (70.4)	1643.6 (1649.4)
		Rest	48.3 (49.0)	1248.8 (1228.8)

Table 4 Percentage of successes for the winner compared to other opinions invented before it till the dominance time, averaged over 10^4 simulation runs for 10 network realizations each when the “speaker-only” agents are randomly selected. The numbers in parentheses correspond to the case of preferential selection of “speaker-only” agents.

N	α		Winner	Earlier opinions
1000	.01	Spo	72.97 (72.68)	2.27 (2.50)
		Rest	81.22 (81.41)	1.63 (1.76)
	.03	Spo	91.57 (95.34)	2.49 (1.64)
		Rest	97.43 (98.65)	0.84 (0.59)
5000	.01	Spo	90.01 (89.92)	1.51 (1.69)
		Rest	95.68 (95.86)	0.65 (0.68)
	.03	Spo	92.1 (93.3)	2.53 (2.48)
		Rest	98.03 (99.05)	0.55 (0.43)

Friendship relations change with due course of time. Hence, it could be interesting to study the effect of rigid individuals embedded on such time-varying social networks.

5.1 Dataset description

For the purpose of the investigation of the naming game dynamics on time-varying networks, we consider two specific real-world face-to-face contact datasets and present our results on each of them. Both the datasets are obtained from SocioPatterns Collaboration (http://www.sociopatterns.org/datasets/). The data collection infrastructure uses active RFID devices embedded in conference badges to detect and store face-to-face proximity relations of persons wearing the badges. These devices can detect face-to-face proximity (1 - 1.5 m) of individuals wearing the badge with a temporal resolution of 20 sec. The first dataset comprises face-to-face interaction data of visitors of the Science Gallery in Dublin, Ireland during the spring of 2009 [23]. This dataset consists of time-varying snapshots of interactions at 20 s time interval for 69 consecutive days. We investigate the time-varying snapshots of one such representative day (22^{nd} day). This network consists of 240 science gallery visitors. For future invocation of this dataset, we shall refer to it as SG22. The other data consists of the conference attendees of ACM Hypertext 2009 held in ISI Foundation in Turin, Italy. The dataset contains the dynamical network of face-to-face proximity of 113 conference attendees over about 2.5 days. For future invocation of this dataset, we shall refer to it as HT.

5.2 The model adaptation in the time-varying setting

The naming game on time-varying network has already been studied by [24] where they play the game in complete synchronization with real time, i.e., at each time step $t = 1, 2 \ldots,$ (the elementary unit of time being second) a game is played among those agents that are alive at that particular instant of time (those agents having degree atleast one) in the network. In this setting, at each time instant, the network snapshot of the agents at that particular time instant is considered. Therefore, this essentially boils down to having a series of network snapshots (one per second) and one game being played on each network snapshot. Please note that, as the RFID device can only consider an interaction if it stays for 20 seconds, the network of agents essentially change after 20 second. This study reports that behavior of the emergent properties of the system for the time-varying case is markedly in contrast with that of the static counterparts. Motivated by the above work, we investigate how the presence of rigid individuals in the population shapes the agreement process on the above real-world dataset. We adopt the same game playing strategy as earlier mentioned with a set of rigid individuals marked before the game starts.

5.3 Results and discussion

The evolution of $N_w(t)$ and $N_d(t)$ on the time-varying graph of HT and SG22 data (see fig 7 (a), (b), (c) and (d)) show a drastically different behavior from the case where these quantities are measured on the static networks. A global consensus usually does not take place on such networks because of their inherent community structure and the openness of the system, i.e., the agents coming in and going out of the system leading to late-stage failures in the system which hinders the consensus (see [24] for further details). Therefore, several opinion clusters get already formed with one becoming the dominant one. The presence of rigid individuals (chosen randomly from the population prior to the beginning of game) breaks such large opinion clusters into several smaller clusters. To analyze this phenomenon further, we observe the frequency of each opinion and find that the dominance of an opinion is not as pronounced in case of time-varying networks (see figs 7(e), (f)) as in static social networks. As one increases α, the size of the clusters tend to become more and more uniform indicating that the frequency of the winner is almost as close as the

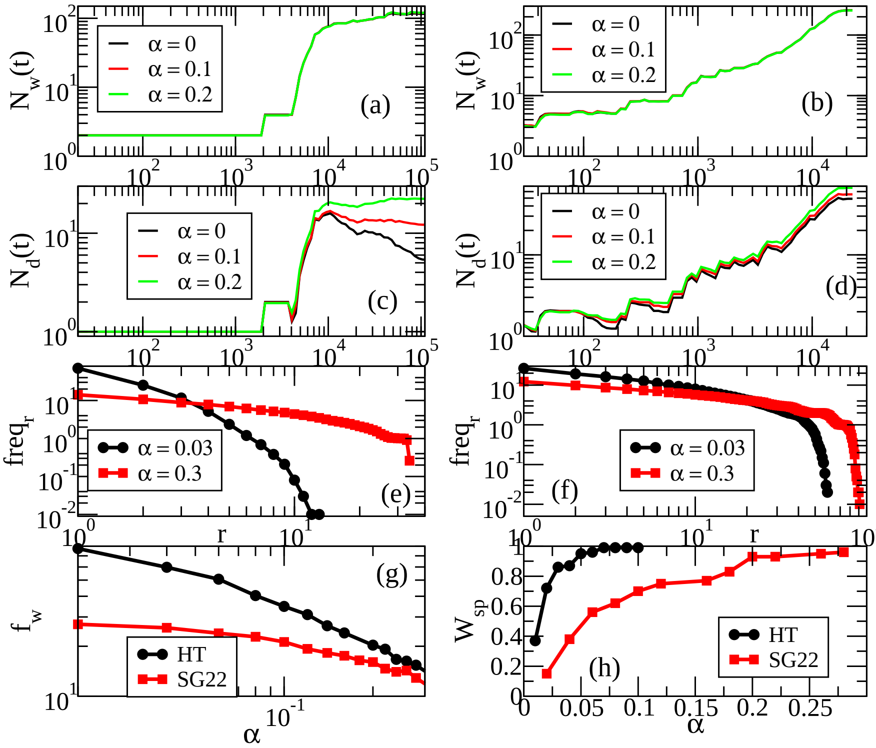

Fig. 7 Time evolution of $N_w(t)$ for (a) HT and (b) SG22 data, $N_d(t)$ for (c) HT and (d) SG22 data. $freq_r$ vs r for (e) HT and (f) SG22 data. (g) f_w vs α for HT and SG22 data. (h) W_{sp} vs α for HT and SG22 data. The curves are averaged over 10^3 simulation runs.

others. Further, we investigate the relationship between the frequency of the winner in the system and α indicating a drastically different behavior as compared to the static networks (see fig 7 (g)). In particular, we do not observe the mirrored S-shaped curve and associated transition; instead, there is a steep drop from the very beginning indicating that these networks already have a strong community structure leading to such multi-opinion states and injecting “speaker-only” agents in the system makes it even more fragmented. Fig 7 (h) shows the relation between the winning probability of the “speaker-only” opinions and the “speaker-only” fraction α. As in case of static networks, here also the 50% winning probability is achieved at a very low value of α.

We further analyze the effect of the presence of “speaker-only” agents on the game interactions. We calculate the occurrence frequency of the number of “speaker-only” agents actively present at different time steps denoted by $size(sp)$ and observe the number of success/failure interactions experienced by $size(sp)$ “speaker-only” agents. The smaller the value of $size(sp)$ the larger is the number of success/failure interactions (success being orders of magnitude higher than failures) as is indicated through figs 8 (a) and (b). This is due to the fact that in majority of the time steps $size(sp)$ is relatively quite low (see figs 8 insets). This indicates that in time-varying networks, it is not only the fraction of “speaker-only” agents that determine the winning opinion but also the number of such agents that are actually actively participating in the social interactions at a time step. The lower is the number of such active participants the higher is the chance that a single dominant opinion emerges.

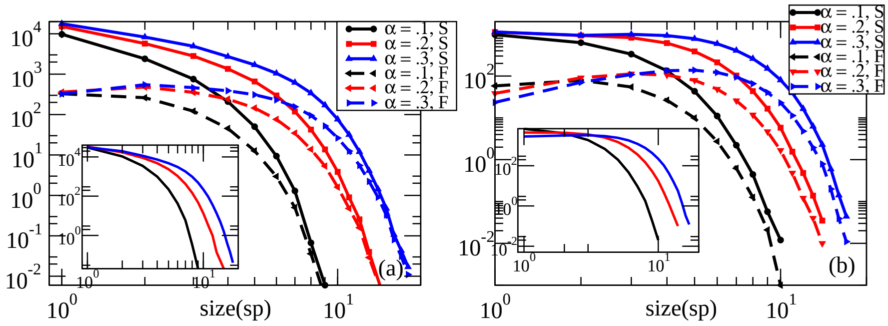

Fig. 8 Number of successes (S) and failures (F) vs $size(sp)$ for (a) HT, (b) SG22 data. Figures in the insets display the occurrence frequencies of different sizes of active “speaker-only” groups for HT and SG22 data respectively. The curves are averaged over 10^3 simulation runs.

6 Conclusions and future works

In conclusion, we have studied the effect of the rigid individuals on the naming game dynamics and how such rigid minorities influence the emergence of a dominant opinion in the system. We observe that the dominance is not the characteristic property of only the “speaker-only” opinions; other opinions from the population may also become dominant. However, with increasing α, the winning probability for a “speaker-only” opinion increases. The dominance index (i.e., f_w) decreases linearly as one increases α and at critical α, it shows an abrupt transition pointing to fragmented state with similar opinion clusters formed around the stubborn opinions. The creation time of an opinion also plays a vital role in the dynamics. Opinions that are invented early in time, possess higher overall chances of becoming the winner except for in the case of “speaker-only” opinions corresponding to static scale-free networks. This observation is quite interesting as this suggest that late-inventors can also produce dominant opinion. This is probably due to the high network heterogeneity in terms of network connectivity. Opinions that are created earlier in time compete among themselves to become the dominant one and the characteristic property of such a dominant opinion is that it takes part in a disproportionately large number of successful interactions (above 80%) compared to its competitors. We have also elucidated the game dynamics on diverse topological structures from homogeneous fully-connected network to heterogeneous scale-free networks and on real world social networks. On static social networks we observe similar results as in case of mean-field, however, for a significantly lower value of α. This indicates that the presence of rigid minorities can strongly affect a society that hardly changes over time. However, if the society is changing fast then such minorities do not seem to have a pronounced effect on the dynamics of opinion formation.

There are quite a few other interesting directions that can be explored in the future. One such direction could be to investigate the effect of introducing a flexibility component of the agents in adopting new opinions (traditionally modeled by a system parameter β that encodes the probability with which the agents update their inventories in case of successful interactions [11]) rather than making them fully rigid. Finally, a thorough analytical estimate of the important dynamical quantities and the cut-off α reported only through empirical evidence here is needed to have a deeper understanding of the emergent behavior of the system.

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