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Emergence of fast agreement in an overhearing population: The case of the naming game

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Abstract – The naming game (NG) describes the agreement dynamics of a population of N agents interacting locally in pairs leading to the emergence of a shared vocabulary. This model has its relevance in the novel fields of semiotic dynamics and specifically to opinion formation and language evolution. The application of this model ranges from wireless sensor networks as spreading algorithms, leader election algorithms to user-based social tagging systems. In this paper, we introduce the concept of overhearing (*i.e.*, at every time step of the game, a random set of N^δ individuals are chosen from the population who overhear the transmitted word from the speaker and accordingly reshape their inventories). When $\delta = 0$ one recovers the behavior of the original NG. As one increases δ, the population of agents reaches a faster agreement with a significantly low-memory requirement. The convergence time to reach global consensus scales as $\log N$ as δ approaches 1.

Introduction. – The naming game (NG) [1] is a simple multi-agent model that employs local communications which leads to the emergence of a shared communication scheme in a population of agents. The game is played by a group of agents in pairwise interactions to negotiate conventions, *i.e.*, associations between forms (names) and meanings (for example individuals in the world, objects, categories, etc.). The negotiation of conventions is a process through which one of the agents (*i.e.*, the speaker) tries to draw the attention of the other agent (the so-called hearer) towards the external meaning by the production of a conventional form. For example, the speaker might be interested to make the hearer identify an object through the production of a name. The hearer may be able to express the proper meaning and the speaker-hearer pair meet a local consensus in which case we call it a "success". The other side of the coin is the hearer producing a wrong interpretation in which case the hearer takes lesson from the meeting by updating her meaning-form association. Thus, on the basis of success and failure of the hearer in producing the meaning of the name, both the interacting agents reshape their internal meaning-form association. Through successive interactions, the local adjustment of the individual meaning-form association leads or should lead to the emergence of a global consensus.

The model represents one of the simplest example leading progressively to the establishment of human-like languages. It was expressly conceived to explore the role of self-organization in the evolution of language [2,3] and it has acquired, since then, a paradigmatic role in the novel field of semiotic dynamics which studies how language evolves through the invention of new words and grammatical constructions, and through adoption of new meaning for words.

This model first proposed in [1], has been rigorously analytically investigated in the mean-field settings in [4] where the authors present explicit derivations for the time required to reach consensus given the size of the population.

Implementing the naming game with local broadcasts, serves as a model for opinion dynamics in large-scale autonomously operating wireless sensor networks. In [5], it is pointed out that NG can be used as a leader election model among a group of sensors where one does not intend to disclose information as to who the leader is at the end of the agreement process. The leader is a trusted agent having possible responsibilities ranging from routing coordination to key distribution and the NG identifies the leader which is hardly predictable from outside resulting in highly secure systems.

The creation of shared classification schemes by the NG in a system of artificial and networked autonomous agents can also be relevant from a system-design viewpoint, *e.g.*, for sensor networks [6,7]. Imagine a scenario where mobile or static sensor nodes are deployed in a large spatially extended region exploring an unknown and possibly hostile environment. One of the important tasks would be to convey information to the agents about their discoveries, in particular they should be able to agree on the identification of the new objects with no prior classification scheme or language to communicate regarding detecting and sensing objects. Since subsequent efficient operation of the sensor network inherently depends on unique object identification, the birth of a communication system among the agents is crucial at the exploration stage after network deployment. Besides artificial systems where it is obvious that the agreement has to take place rapidly, it concerns social dynamics too [8]. In particular, as an example, one can think of the emergence of shared lexicon inside social groups and communities. When a new concept is introduced, different people name it differently. These words spread among the population, competing against each other, until the choice of one of them is adopted and everybody uses the same word [9–11]. This type of dynamics is very effective in modeling the behavior of social groups and communities involved in user-based tagging systems (such as flickr.com or del.icio.us) [12,13], where users manage tags to share and categorize information as well as in activities such as the "likes" of Facebook[1] and Twitter[2].

The minimal NG has diverse applications in many fields [5–7,12,13] and been studied on various topology of networks apart from the fully connected network. The model, for instance, is studied in regular lattices [14,15]; small world networks [15–18]; random geometric graphs [15,19,20]; and static [21–23], dynamic [24], and empirical [25] complex networks. The final state of the system is usually a complete consensus [26], but stable polarized states can be reached introducing a simple confidence/trust parameter [27]. This simple model has also been modified in several ways [15,19,25,27–36] and it represents the fundamental stepping stone of more complex models in computational cognitive sciences [37–40].

Here in this paper, we shall reshape the model in a "multi-party" communication framework. In particular, this involves conversations between two parties and plays a significant role in the formation of shared mental model [41]. Parties involved in a multi-party dialogue can assume roles other than the speaker/addressee roles in traditional two-party communication. One of the most important roles is that of the *overhearer*. Overhearing involves monitoring the routine conversations of agents, who know they are being overheard, to infer information about the agents. The overhearers might then use such information to assist themselves, assess their individual progress or suggest advice to the others. When an agent "overhears an interaction", she receives information about something that is not primarily addressed to her. For instance, one can listen to a conversation between two friends without being part of their dialogue. Multi-party discourse analysis shows that overhearing is a required communication type to model group interactions and consequently reproduces them among artificial agents [42]. Various applications are known to employ the concept of overhearers [43–46]. Novick and Ward [43] have employed overhearing to model interactions between pilots and air traffic controllers. Aiello *et al.* [44] and Bussetta *et al.* [45] have investigated an architecture that enables overhearing, so that domain experts can provide advice to problem-solving agents when necessary. Legras [46] has examined the use of overhearing for maintaining organizational awareness. Recently, Komarova *et al.* [47] have studied the effect of eavesdropping in the evolution of language.

Motivated by the above literature and diverse applications of overhearer, we review the naming game for the emergence of a communication system in the presence of overhearers and attempt to investigate its global properties. To the best of our knowledge, NG has not been studied in this perspective of multi-party communication. The basic activity of the overhearers in the naming game is as follows: when a conversation between two parties is going on, the third party (*i.e.*, the overhearers) may eavesdrop the conversation and reshape their meaning-form association. As we shall see in this paper that the introduction of the concept of overhearing leads to much faster convergence than traditional NG [4] coupled with a low-memory requirement per agent. The method of broadcasting in naming game as presented in [5] is a very close correlate of our scheme especially when the set of overhearers constitute the entire population except the speaker-hearer pair. For $N \to \infty$ and $\delta > 0$, the number of overhearers in our model diverges, while the number of speaker is always fixed at one. Thus, the only difference between the overhearing population and the broadcasting scheme is the update rule of the speaker which is shown to be negligible in [5] for the HO-NG (Hearer-Only NG) scheme.

This overhearing model can also be suitably applied to the describe the process of rumor spreading [48–56]. The area of rumor spreading has a rich history and a variety of models exist in literature. One of the benchmark method has been the PUSH-PULL strategy [51–56]. The simple PUSH-PULL mechanism is as follows: at each round, a node that knows the rumor selects a random neighbor and forwards the rumor (PUSH), or if the node does not know the rumor selects a neighbor uniformly at random and asks for the information (PULL). This scheme informs all N agents in a fully connected network in time $\log_3 N + O(\ln\ln N)$ with probability at least $1 - O(N^{-\alpha})$

[1] `http://www.facebook.com/`.
[2] `http://twitter.com/`.

where $\alpha > 0$. The problem has been studied on networks with conductance ϕ in [55,56]. In particular, the authors achieve a tight bound on the number of rounds required in spreading a rumor over a connected network of N nodes and conductance ϕ which is $O(\frac{\log N}{\phi})$. Estimates regarding the amount of memory required per agent for the purpose of spreading have not been presented so far in the above literature. In general, since there is usually one rumor to be spread the memory estimate becomes trivial ($O(1)$). However, one can always envisage a situation where new rumors need to be constantly invented in the population.

The rest of the article is organized as follows. The second section is devoted to the description of the basic naming game model in the presence of overhearers. In the third section, we investigate the scaling relations of some important quantities and provide analytical arguments to derive the relevant exponents. In the fourth section, we discuss the state of the art and compare our findings with [56]. Finally, conclusions are drawn in the fifth section.

The model definition. – The model consists of an interacting population of N artificial agents observing a single object to be named, *i.e.*, a set of form-meaning pairs (in this case only names competing to name the unique object) which is empty at the beginning of the game ($t = 0$) and evolves dynamically in time. At each time step ($t = 1, 2, \ldots$) two agents are randomly selected and interact: one of them plays the role of *speaker*, the other one that of *hearer*. In addition, a set of N^δ individuals are randomly selected in each step who behave as *overhearers*. Note that δ is a parameter of the model.

In each game the following steps are executed:

- The speaker transmits a name to the hearer. If her inventory is empty, the speaker invents a new name, otherwise she selects randomly one of the names she knows.
- If the hearer has the uttered name in her inventory, the game is a success, and both agents delete all their names, but the winning one.
- If the hearer does not know the uttered name, the game is a failure, and the hearer inserts the name in her inventory.
- Each overhearer overhears the word uttered by the speaker; if the word is in her inventory, she removes all the words from her inventory except this word (*i.e.*, treats the event as a success) else she adds this word in her inventory (*i.e.*, treats the event as a failure).

Figure 1 shows a hypothetical example illustrating the inventory update rules of the different agents in the model of NG with overhearers.

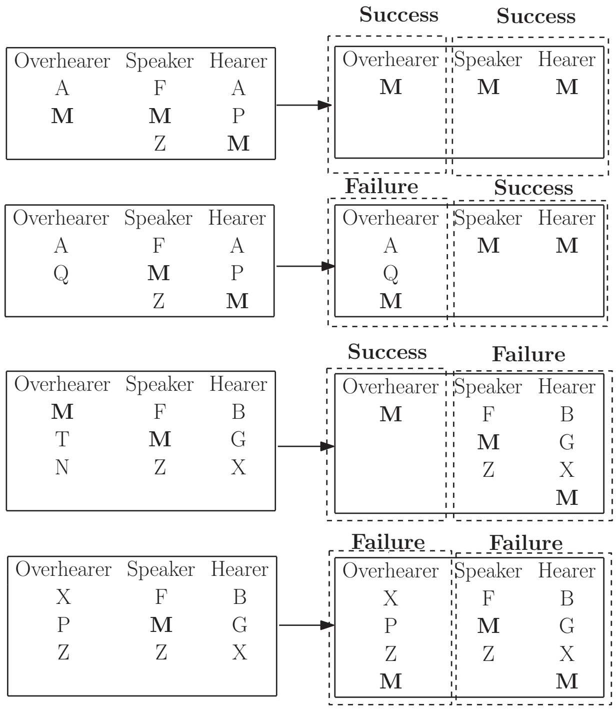

Fig. 1: Naming game interaction rules in presence of overhearer. The names are denoted symbolically in English alphabet and boldface signifies the name that the speaker transmits to the hearer.

Results and discussions. – The basic quantities to be measured in the NG are the total number of words $N_w(t)$, defined as the sum of the inventory sizes of all the agents at the given time instance t, and the number of different words $N_d(t)$ present in the system at time t, telling us how many synonyms are present in the system at that time instance. The dynamics proceeds as illustrated in fig. 2(a), (b), (c) and (d). At the beginning both $N_w(t)$ and $N_d(t)$ grow linearly as the agents invent new words. As invention ceases, $N_d(t)$ reaches a plateau, *i.e.*, a maximum number of distinct words. On the other hand, $N_w(t)$ keeps growing till it reaches a maximum at time t_{max}. The total number of words then decreases and the system reaches the convergence state at time t_{conv}. At convergence all the agents share the same unique word, so that $N_w(t_{conv}) = N$ and $N_d(t_{conv}) = 1$. It is observed that all the global quantities in the basic naming game [1] follow a power-law scaling as a function of the population size N. In particular, $t_{max} \sim N^\alpha$, $t_{conv} \sim N^\beta$, $N_w^{max} \sim N^\gamma$ where $\alpha \approx \beta \approx \gamma \approx 1.5$ for the original naming game ($\delta = 0$) on a fully connected graph topology.

We now focus on analytically estimating the scaling of i) N_w^{max}, ii) t_{max} and iii) t_{conv} with N in the presence of N^δ overhearers.

Scaling of N_w^{max}. In the original NG the maximum number of distinct words scales as N with an average value of $N/2$. This is because at each time step only two agents can update their inventories, inventing in particular a new word if their inventories are empty. When N^δ overhearers are present the fraction of agents who can invent new words is reduced by a factor N^δ. In this way the number of unique words in the system when the total number of words is close to the maximum is

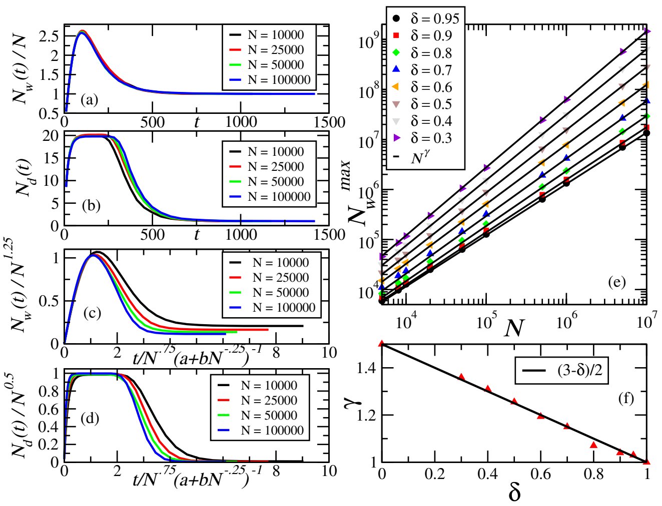

Fig. 2: (Colour on-line) Evolution of $N_w(t)$ when the number of overhearers is (a) ηN with $\eta = 0.05$ (c) N^δ where $\delta = 0.5$ for different values of N. Time evolution of $N_d(t)$ when the number of overhearers is (b) ηN with $\eta = 0.05$ (d) N^δ where $\delta = 0.5$ for different values of N. (e) Scaling of N_w^{max} with N for different values of δ. (f) The figure expresses the relation of γ *vs.* δ. Each point in the above curves represents the average value obtained over 100 simulation runs.

$\propto N/N^\delta = N^{1-\delta}$. Further, let us assume that each agent has on an average cN^a words in her inventory when the total number of words is close to the maximum. As in the original NG, $N_w^{max} \sim N^\gamma$ so that $\gamma = a+1$ holds here also. In the following, we shall attempt to find a relation between γ and δ. We can write the evolution equation of $N_w(t)$ as

$$\frac{\mathrm{d}N_w(t)}{\mathrm{d}t} \propto \left(1 - \frac{cN^a}{N^{1-\delta}}\right) N^\delta - \frac{cN^a}{N^{1-\delta}} cN^a N^\delta, \qquad (1)$$

where the first term is related to unsuccessful games (increase in N_w is proportional to N^δ times the probability of a single failure) and the second term is for successful games (decrease in N_w is proportional to cN^aN^δ times the probability of a single success). At maximum, $\frac{\mathrm{d}N_w(t_{max})}{\mathrm{d}t} = 0$ and therefore in the limit $N \to \infty$ the only relation possible is $a = \frac{1-\delta}{2}$ which implies $\gamma = \frac{3-\delta}{2}$. When $\delta = 0$, $\gamma = 1.5$ we recover the original NG behavior. In general, as one varies δ in the interval $[0,1)$, N_w^{max} varies as N^γ where $\gamma \in [1, 1.5]$. The scaling of N_w^{max} with δ for different values of N is shown in fig. 2(a) and (c) through a data collapse and in fig. 2(e) through curve fitting. In fig. 2(a) and (c), we present a curve rescaling of $N_w(t)$. We rescale $N_w(t)$ by N_w^{max} and t by t_{max} (calculated in the next section). Similarly, for $N_d(t)$ we perform data collapse by rescaling $N_d(t)$ by N_d^{max} and t by t_{max} (see fig. 2(b) and (d)). This rescaling results in a perfect overlap of the quantities N_w^{max} and t_{max} for different N. Note that it is not possible to indicate the collapse of t_{conv} in the same figures and hence we refer the reader to fig. 5 for results related to t_{conv}. For all values of N, N_w^{max} monotonically decreases as δ increases and in the limit $\delta \to 1$ we have

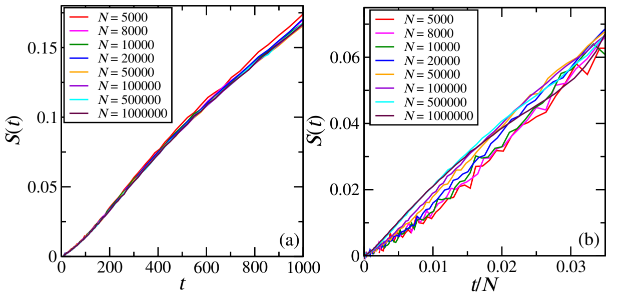

Fig. 3: (Colour on-line) Success rate at the onset of the dynamics. (a) Success rate $S(t) \propto t$ when the number of overhearers $= \eta N$, where $\eta = 0.05$. (b) $S(t) \propto t/N$ when $\delta = \frac{1}{2}$. All the curves have been generated averaging over 100 simulation runs.

$N_w^{max} \to N$. This behavior of γ *vs.* δ is confirmed by the simulation results shown in fig. 2(f).

Scaling of t_{max}. We have to analyze the behavior of the success rate in the beginning of the process in order to estimate the scaling relations for t_{max}. At early stages, most successful interactions involve agents which have already met in previous games. Thus, the probability of success is proportional to the ratio between the number of couples that have interacted before time t, which is $\propto tN^\delta(N^\delta - 1)/2$ and the total number of possible pairs is $N(N-1)/2$. Thus, in the early stages, the success rate $S(t) \propto \frac{tN^{2\delta}}{N^2} = tN^{2(\delta-1)}$. Note that if we put $\delta = 0$, we immediately recover $S(t) \propto t/N^2$ which is the case for the original NG. If $\delta \to 1$, we have $S(t) \propto t$, while if $\delta = \frac{1}{2}$ we have $S(t) \propto t/N$. Both these observations are validated by fig. 3(a) and (b), respectively, for different values of N.

With this information about $S(t)$ we can now easily estimate the value of t_{max} by once again writing the evolution equation:

$$\frac{\mathrm{d}N_w(t)}{\mathrm{d}t} \propto \left(1 - tN^{2(\delta-1)}\right) N^\delta - tN^{2(\delta-1)} cN^{(1-\delta)/2} N^\delta. \qquad (2)$$

If we now impose $\frac{\mathrm{d}N_w(t_{max})}{\mathrm{d}t} = 0$, then in the limit $N \to \infty$ we have $t_{max} \propto \frac{N^{\frac{3(1-\delta)}{2}}}{a+bN^{-(1-\delta)/2}}$ where the denominator is precisely a correction term with a and b as constants. Once again, for $\delta = 0$ we have $t_{max} \propto N^{3/2}$ thus recovering the original NG property. On the other hand, in the limit $\delta \to 1$, t_{max} approaches $O(1)$. The results of the scaling of t_{max} with N for different values of δ are shown in fig. 4.

Scaling of t_{conv}. The exponent for the convergence time, β, deserves a more intricate discussion, and we can only attempt to provide a naïve argument here. We concentrate on the scaling of the interval of time separating the peak of $N_w(t)$ and the convergence, *i.e.*, $t_{diff} = (t_{conv} - t_{max})$, since we already have an argument for t_{max}. t_{diff} is the time span required by the system to get rid of all the words but the one which survives in the final state.

If we adopt the mean-field assumption that at $t = t_{max}$ each agent has on average $N_w^{max}/N \sim N^{\frac{1-\delta}{2}}$ words, we see

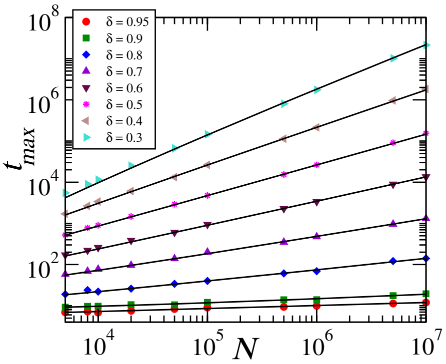

Fig. 4: Scaling of t_{max} with population size N. As one varies δ, t_{max} scales as $\frac{N^{\frac{3(1-\delta)}{2}}}{a+bN^{(\delta-1)/2}}$ where a and b are some constants. Each data point of all the above curves represents the average value taken over 100 simulation runs. The bold lines show the fit from the analytical results.

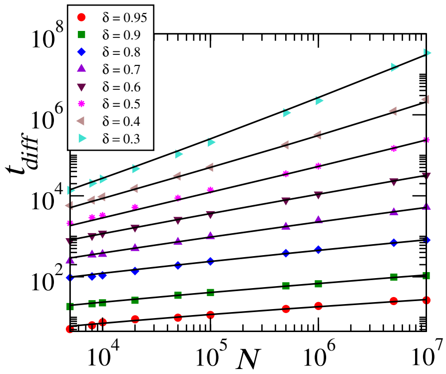

Fig. 5: (Colour on-line) Scaling of t_{diff} with the population size N. As δ varies, this interval time t_{diff} scales as $N^{\frac{3(1-\delta)}{2}}(a+bN^{(\delta-1)/2})\log N$ where a and b are some constants. Each point represents the average value obtained from 100 simulation runs. The bold lines show the fit from the analytical results.

that, by definition, in the interval t_{diff}, each agent must have won at least once. This is a necessary condition to have convergence, and it is interesting to investigate the timescale over which this happens. Assuming that $\overline{N}$ is the number of agents who did not yet have a successful interaction at time t, we have

$$\overline{N} = N(1-p_s p_w)^t, \tag{3}$$

where p_s is the probability of choosing a specific agent and $p_w = S(t)$ is the probability of a success. In this case, $p_s = \frac{1}{N^{1-\delta}}$ and $p_w = tN^{2(\delta-1)}$. In order to estimate t_{diff}, we require the number of agents who have not yet had a successful interaction to be finite just before the convergence, *i.e.*, $\overline{N} \sim O(1)$ and we consider $p_w(t_{max}) = t_{max}N^{2(\delta-1)} = \frac{N^{-(1-\delta)/2}}{a+bN^{-(1-\delta)/2}}$. In this way one gets

$$t_{diff} \propto N^{\frac{3(1-\delta)}{2}}(a+bN^{-(1-\delta)/2})\log N. \tag{4}$$

The above scaling relation of t_{diff} is well confirmed by the simulation results in fig. 5.

Thus, when $\delta = 0$, and we ignore the correction, we recover the original NG case: $t_{conv} \propto N^{3/2}\log N$. On the other hand, in the limit $\delta \to 1$, we have $t_{conv} \to \log N$. This logarithmic scaling is possibly due to the fact that to reach consensus, all individuals must participate at least once and hence would need a very small time before convergence could be reached. For $\delta \to 1$, the convergence therefore is almost instantaneous.

Conclusions and future work. – In this paper, we have introduced the agreement dynamics of naming game to describe the convergence of population of agents on assigning a unique name to an object in the domain of multi-party communication. We have investigated the basic naming-game model in an overhearing population and computed the scaling behaviour of the main global quantities: $N^w_{max} \propto N^\gamma$ where the exponent $\gamma = \frac{3-\delta}{2}$; $t_{max} \propto N^\alpha$ where roughly the exponent $\alpha = \frac{3(1-\delta)}{2}$ and $t_{conv} \propto N^\alpha \log N$. In particular, we achieve a very fast agreement in the population with significantly low-memory requirement. Moreover, we have also suggested that this model with overhearers can find relevant application in rumour spreading. We show, theoretically as well as by means of simulations, that rumor spreading in a fully connected network (*i.e.*, conductance $\phi = 1$) of N nodes takes a $O(\log N)$ time to reach the global agreement with a maximum memory estimate of N as $\delta \to 1$ which is comparable with the time requirement for the spreading of rumor in [54,56]. It is important to stress that in our model the case of maximal conductance $\phi = 1$ is obtained on a fully connected graph only in the limit $\delta = 1$. Further, we point out that the model of NG in overhearing population can be recast for rumor spreading when constantly new rumors get generated which should compete with each other to spread in the whole population.

There could be many interesting future directions. First of all, it will be interesting to explore the model in a scenario where agents make their success update probabilistically as studied in [27]. The role of topology in the dynamics is also one of the important future perspectives. Different complex topologies could be studied where agents are embedded on more realistic networks representing social relations. Furthermore, while in this paper we have concentrated only on the study of the scaling properties of the system, performing a detailed analysis of the microscopic aspects of the dynamics could be another interesting topic for future research. One might also extend the idea of overhearers to more complex tasks like categorization [37–40].

REFERENCES

[1] Baronchelli A., Felici M., Loreto V., Caglioti E. and Steels L., *J. Stat. Mech.* (2006) P06014.
[2] Steels L., *Artif. Life*, **2** (1995) 319.

[3] STEELS L., *Self-organizing vocabularies*, in *Proceedings of Artificial Life V* (Nara, Japan) 1996, pp. 179–184.

[4] BARONCHELLI A., LORETO V. and STEELS L., *Int. J. Mod. Phys. C*, **19** (2008) 785.

[5] BARONCHELLI A., *Phys. Rev. E*, **83** (2011) 046103.

[6] LEE Y., COLLIER T. C., TAYLOR C. E. and STABLER E. P., *The role of population structure in language evolution*, presented at *AROB'05*.

[7] COLLIER T. C. and TAYLOR C., *J. Parallel Distrib. Comput.*, **64** (2004) 866.

[8] CASTELLANO C., FORTUNATO S. and LORETO V., *Rev. Mod. Phys.*, **81** (2009) 591.

[9] LASS R., *Historical Linguistics and Language Change* (Cambridge University Press) 1997.

[10] BRISCOE E. J. (Editor), *Linguistic Evolution through Language Acquisition: Formal and Computational Models* (Cambridge University Press) 2002.

[11] HURFORD J., STUDDERT-KENNEDY M. and KNIGHT C. (Editors), *Approaches to the Evolution of Language: Social and Cognitive Bases* (Cambridge University Press) 1998.

[12] CATTUTO C., LORETO V. and PIETRONERO L., *Proc. Natl. Acad. Sci. U.S.A.*, **104** (2007) 1461.

[13] GOLDER S. and HUBERMAN B. A., *J. Inf. Sci.*, **32** (2006) 198.

[14] BARONCHELLI A., DALL'ASTA L., BARRAT A. and LORETO V., *Phys. Rev. E*, **73** (2006) 015102.

[15] LU Q., KORNISS G. and SZYMANSKI B. K., *Phys. Rev. E*, **77** (2008) 016111.

[16] DALL'ASTA L., BARONCHELLI A., BARRAT A. and LORETO V., *Europhys. Lett.*, **73** (2006) 969.

[17] LIU R.-R., JIA C.-X., YANG H.-X. and WANG B.-H., *Physica A*, **388** (2009) 3615.

[18] BARRAT A., BARONCHELLI A., DALL'ASTA L. and LORETO V., *Chaos*, **17** (2007) 026111.

[19] LU Q., KORNISS G. and SZYMANSKI B. K., arXiv:cs/0604075 [cs.MA] (2006).

[20] JIA-BO H., HAN-XIN Y., RUN-RAN L., BING-HONG W. and ZHI-YUAN Z., *Chin. Phys. Lett.*, **27** (2010) 090202.

[21] DALL'ASTA L., BARONCHELLI A., BARRAT A. and LORETO V., *Phys. Rev. E*, **74** (2006) 036105.

[22] DALL'ASTA L. and BARONCHELLI A., *J. Phys. A: Math. Gen.*, **39** (2006) 14851.

[23] YANG H.-X., WANG W.-X. and WANG B.-H., *Phys. Rev. E*, **77** (2008) 027103.

[24] NARDINI C., KOZMA B. and BARRAT A., *Phys. Rev. Lett.*, **100** (2008) 158701.

[25] LU Q., KORNISS G. and SZYMANSKI B., *J. Econ. Interact. Coord.*, **4** (2009) 221.

[26] VYLDER B. D. and TUYLS K., *J. Theor. Biol.*, **242** (2006) 818.

[27] BARONCHELLI A., DALL'ASTA L., BARRAT A. and LORETO V., *Phys. Rev. E*, **76** (2007) 051102.

[28] WANG W. X., LIN B. Y., TANG C. L. and CHEN G. R., *Eur. Phys. J. B*, **60** (2007) 529.

[29] BRIGATTI E., *Phys. Rev. E*, **78** (2008) 046108.

[30] LIPOWSKI A. and LIPOWSKA D., *Int. J. Mod. Phys. C*, **19** (2008) 399.

[31] LIPOWSKI A. and LIPOWSKA D., *Phys. Rev. E*, **80** (2009) 056107.

[32] BRIGATTI E. and RODITI I., *New J. Phys.*, **11** (2009) 023018.

[33] LEI C., WU T., WANG L. and JIA J.-Y., *Physica A*, **389** (2010) 4046.

[34] ZHANG W. and LIM C. C., arXiv:1008.4115 [cs.MA] (2010).

[35] YONG Z., RUN-RAN L., HAN-XIN Y., DONG-MEI S. and BING-HONG W., *Chin. Phys. Lett.*, **27** (2010) 108901.

[36] LEI C., JIA J., WU T. and WANG L., *Physica A*, **389** (2010) 5628.

[37] PUGLISI A., BARONCHELLI A. and LORETO V., *Proc. Natl. Acad. Sci. U.S.A.*, **105** (2008) 7936.

[38] BARONCHELLI A., GONG T., PUGLISI A. and LORETO V., *Proc. Natl. Acad. Sci. U.S.A.*, **107** (2010) 2403.

[39] MUKHERJEE A., TRIA F., BARONCHELLI A., PUGLISI A. and LORETO V., *PLoS ONE*, **6** (2011) e16677.

[40] LORETO V., MUKHERJEE A. and TRIA F., *Proc. Natl. Acad. Sci. U.S.A.*, **109** (2012) 6819.

[41] KAMALI K., FAN X. and YEN J., *Multiparty proactive communication: a perspective for evolving shared mental models*, in *Proceedings of AAAI'06* (AAAI Press) 2006, pp. 685–690.

[42] DIGNUM F. and VREESWIJK G., *Towards a testbed for multi-party dialogues*, in *Advances in Agent Communication*, edited by DIGNUM F., Vol. **2922** (Springer, Berlin) 2004.

[43] NOVICK D. G. and WARD K., *Mutual beliefs of multiple conversants: a computational model of collaboration in air traffic control*, in *Proceedings of AAAI'93* (AAAI Press) 1993, pp. 196–201.

[44] AIELLO M., BUSETTA P., DONÀ A. and SERAFINI L., *Ontological overhearing*, in *Proceedings of ATAL'01* (Springer Verlag) 2002, pp. 175–189.

[45] BUSETTA P., SERAFINI L., SINGH D. and ZINI F., *Extending multi-agent cooperation by overhearing*, in *Proceedings of CooplS'01* (Springer Verlag) 2001, pp. 40–52.

[46] LEGRAS F. and TESSIER C., *Lotto: group formation by overhearing in large teams*, in *Proceedings of AAMAS'03* (ACM) 2003, pp. 425–432.

[47] KOMAROVA N. L. and LEVIN S. A., *J. Theor. Biol.*, **264** (2010) 104.

[48] BERG S., *J. Appl. Probab.*, **20** (1983) 31.

[49] DALEY D. J. and KENDALL D. G., *IMA J. Appl. Math.*, **1** (1965) 42.

[50] MAKI D. P. and THOMPSON M., *Mathematical Models and Applications: With Emphasis on the Social, Life, and Management Sciences* (Prentice-Hall) 1973.

[51] FRIEZE A. and GRIMMETT G., *Discrete Appl. Math.*, **10** (1985) 57.

[52] PITTEL B., *SIAM J. Appl. Math.*, **47** (1987) 213.

[53] DEMERS A., GREENE D., HAUSER C., IRISH W., LARSON J., SHENKER S., STURGISN H., SWINEHART D. and TERRY D., *Epidemic algorithms for replicated database maintenance*, in *Proceedings of PODC '87* (ACM) 1987, pp. 1–12.

[54] KARP R., SCHINDELHAUER C., SHENKER S. and VOCKING B., *Randomized rumor spreading*, in *Proceedings of FOCS'00* (IEEE Computer Society) 2000, pp. 565–574.

[55] CHIERICHETTI F., LATTANZI S. and PANCONESI A., *Rumour spreading and graph conductance*, in *Proceedings of SODA'10* (SIAM) 2010, pp. 1657–1663.

[56] CHIERICHETTI F., LATTANZI S. and PANCONESI A., *Almost tight bounds for rumour spreading with conductance*, in *Proceedings of STOC'10* (ACM) 2010, pp. 399–408.