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Understanding how dominance affects the emergence of agreement in a social network: The case of Naming Game

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***Abstract*—We study the dynamics of the Naming Game as an opinion formation model on social networks. This agent-based model captures the essential features of the agreement dynamics by means of a memory-based negotiation process. Our study focuses on the impact of dominance of certain opinions over others in pursuit of faster agreement on social networks. We propose two models to incorporate dominance of the opinions. We observe that both these models lead to faster agreement among the agents on an opinion as compared to the base case reported in the literature. We perform extensive simulations on computer-generated networks as well as on a real online social network (Facebook) and in both cases the dominance based models converge significantly faster than the base case.**

I. INTRODUCTION

Social networks exhibit a wide range of interaction pattern and a variety of "actors" are engaged in such interactions. Not all the actors are similarly influential or dominant. Some are more opinionated and while others tend to be more passive. Therefore, there is a dominance factor involved in almost any social interaction. Under normal circumstances, the highly influential actors in a social network would tend to influence the opinions of the rest of the "not-so-influential" actors. This aspect, we believe, is extremely important in the formulation of any dynamical model of a societal system.

In this paper, we focus on the popular Naming Game model (NG) [1] as a model of opinion formation to study how social dominance influence opinion spreading and how societies move towards consensus in the adoption of a single opinion through negotiation. This is a simple multi-agent model that employs local communications which leads to the emergence of a shared communication scheme/common opinion in a population of agents. The system evolves through local pairwise interactions among artificial agents that necessarily capture the generic and essential features of an agreement process. This model was conceived to explore the role of self-organization in the evolution of languages [2], [3] and has acquired a paradigmatic role in semiotic dynamics that studies evolution of languages through invention of new words, grammatical constructions and more specifically, through adoption of new meaning for different words. NG finds widespread applications in various fields ranging from artificial sensor networks as a leader election model [4] to the social media as an opinion formation model.

The minimal Naming Game (NG) consists of a population of N agents observing a single object in the environment (may be a discussion on a particular topic) and opining for that by means of communication to one another through pairwise interactions, in order to reach a global agreement. The agents have at their disposal an internal inventory, in which they can store an unlimited number of different words or opinions. At the beginning, all the individuals have empty inventories. At each time step, the dynamics consists of a pairwise interaction between randomly chosen individuals. The chosen individuals can take part in the interaction as a "speaker" or as a "hearer." The speaker voices to the hearer a possible opinion for the object under consideration; if the speaker does not have one, i.e., his inventory is empty, he invents an opinion. In case where he already has many opinions stored in his inventory, he chooses one of them randomly. The hearer's move is deterministic: if she possesses the opinion pronounced by the speaker, the interaction is a "success", and both speaker and hearer retain that opinion as the right one, removing all other competing opinions/words from their inventories; otherwise, the new opinion is included in the inventory of the hearer, without any cancellation of opinions in which case the interaction is termed as a "failure" (see fig 1). The game is played on a fully connected network, i.e., each agent can, in principle, communicate with all the other agents, and makes the following basic assumption. In the invention process, the number of possible invented opinions is so large that the probability of an opinion being reinvented is practically negligible (this means that similar opinions are not taken into account here, although the extension is trivially possible). As a consequence, one can reduce, without loss of generality, the environment to be consisting of only one single object/topic of discussion.

There has been a lot of research on opinion dynamics and a wide variety of opinion formation models exist in literature [5], [6]. In fact, NG itself has been studied as an opinion formation model in [7], [8] and different variants of the model have been investigated so far in [4], [9]–[12]. Dominance of individuals is an important aspect in opinion dynamics which can significantly influence the outcome of the dynamics. But to the best of our knowledge, this type of agent dominance has not been studied so far in the literature. In this paper,

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Fig. 1. Agent's interaction rules in basic NG. Suppose there is a topic on which a discussion is going on, say "Who is the best chess player?". (Top) The speaker chosen at random, opines for "Fischer" (also chosen randomly from his inventory of opinions). Now, the hearer (again chosen at random) does not have this opinion in her inventory, and therefore she adds the opinion "Fischer" in her inventory and the interaction is a failure . (Bottom) The speaker opines for "Kasparov" and in this case the opinion is present in the hearer's inventory. So, they delete all other opinions except "Kasparov". The interaction this time is "success".

we investigate how dominance, if already present in a society in some form, can influence the overall agreement dynamics for opinion formation. Note that another important direction could be to investigate the phenomenon of the emergence of dominance itself; however, in this study we assume that dominance is already existing in the society modeled through certain popularity metrices of the agents. We formulate two forms of dominance. One is the degree (size of the social neighborhood) of an agent indicating its popularity. The other one is the success rate of the agent indicating how successful one is in propagating opinions. We observe that the opinions of high dominance force the low-dominant opinions to get disposed off the system quickly, keeping the competition for survival alive only to the dominant lot, thereby, resulting in a faster agreement in the population. This fast agreement is observed irrespective of the underlying societal structure. In fact, the most advanced model reaches consensus in time that scales almost linearly as compared to the classical case where a super-linear behavior is observed.

The rest of the paper is organized as follows. In section 2, we propose two variants of the Naming Game based on dominance. Section 3 provides the elaborate impact of our proposed model for different social structures and points to possible explanations of our findings. Finally, conclusions are drawn in section 4.

II. MODELING DOMINANCE IN NG

In this section, we shall attempt to model dominance and couple it with the minimal Naming Game. The dominance of an individual is reflected through his/her dominating opinion which other individuals tend to follow. In the process of agreement, conforming to the opinion of the dominant individual might be a better option when we do not have much information on the topic of discussion. On the other hand, the dominant individuals try to influence the individuals low in confidence in adopting their opinions. Thus, the most natural way of modeling the dominance could be associating weights to the opinion proportional to the degree of the inventor of that opinion (i.e., the number of other agents connected to the inventor) as degree is a natural reflection of the topological dominance of a particular individual in a social network. Now, as the game evolves, the opinions get rewarded with increase in weights through successful interactions among individuals. Therefore, the dominance of an opinion dynamically evolves as it takes part in more and more successful events. Below, we describe the modified Naming game with the dominance taken into account.

The evolutionary rules are as follows:

- At each time step (t = 1, 2, ..), a speaker i is chosen at random and then the speaker chooses one of his neighbors as the hearer j.
- If the speaker i's inventory is empty, he invents a new opinion and assigns a weight k_i to the opinion, where k_i is the degree of i. Otherwise, if i already knows one or more opinions for the object, then an opinion l is chosen using the preferential rule, with the probability
$$p_l = \frac{w_l}{\sum_{l=1}^{n} w_l}$$
where w_l is the weight of the l^{th} opinion in the speaker's inventory and n is the number of opinions in speaker's inventory. The invented or selected opinion is then communicated to the hearer along with its weight.
- If the hearer j has the opinion in her inventory irrespective of its weight, the communication is a "success" and both of them keep this opinion and delete other opinions from their inventories. Both the speaker and the hearer increase the weight of the opinion by 1.
- If the hearer j does not have the opinion in her inventory, the communication is termed as "failure" and the hearer adds this opinion into her inventory along with its weight. (see fig 2)

In future invocation of this model, we shall refer to it as NGD.

However, when we do not get any benefit if the topological structure is regular and then the dominance is determined only by the evolutionary factor. An opinion, in this case, becomes more dominant as it takes part in more successful interactions. Thus, we propose another variant of dominance model that takes into account only the evolutionary dominance. The necessary modifications are as follows:

- When a speaker invents an opinion, we associate a very small weight ϵ to the opinion which is uniform for all inventions. This ϵ can be a parameter of the model.
- With success events, the weight associated with the successful opinion increases in both the speaker and hearer's inventory and on failure, the hearer copies the opinion along with its weight.

For future invocation of this model in this paper, we shall refer to it as NGS.

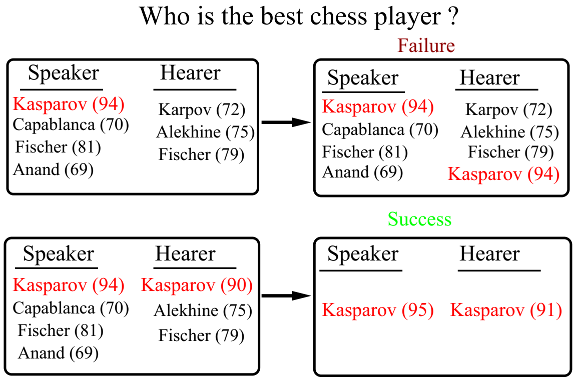

Fig. 2. Agent's interaction rules in the dominant model. The numbers within parenthesis refer to the weight associated with the opinions and an opinion is chosen proportional to its weight. So, the opinion "Kasparov" has the highest probability of being chosen and assume that it is chosen. The hearer doesn't have the opinion and hence a failure. On failure, the hearer copies the opinion with its weight. On success (Bottom), the speaker and hearer increases the weight of the successful opinion by 1.

The main quantities of interest which describe the emergent properties of the system are

- the total number $N_w(t)$ of words/opinions in the system at the time t (i.e., the total size of the memory);
- the number of different (i.e., unique) words/opinions $N_d(t)$ in the system at the time t;
- the average success rate $S(t)$, i.e., the probability, computed averaging over many simulation runs, that the chosen agent gets involved in a successful interaction at a given time t.

The system reaches the global consensus described by the following state $N_w(t) = N$, $N_d(t) = 1$ and $S(t) = 1$ where N is the size of the population. Apart from the basic quantities describing the system, the quantities which are of interest from the global perspective are the time to reach the consensus (t_{conv}), the maximum memory required by the agents during the process (N_w^{max}) and the time required to reach the memory peak (t_{max}).

III. RESULTS AND DISCUSSION

In this section, we perform a comparative analysis of the dominance based Naming Game with the minimal Naming Game.

A. The Mean-field case

The mean-field case corresponds to a fully connected network in which all agents are in mutual contact. Thus, every individual can, in principle, talk to every other individual. The perfectly regular structure prevents any single individual from being more influential by taking advantage of the degree based skew. Every individual therefore are equally dominant and hence the NGS model is perfectly suitable for the scenario.

In fig 3, we compare the basic emergent properties of the system of NGS with the minimal NG. The evolution of $N_w(t)$ curve shows that the NGS reaches agreement faster with a lower memory peak and a sharper drop of memory size in the post-N_w^{max} phase of the dynamics. The time evolution of $N_d(t)$ shows a pretty similar behavior in both the models in pre-N_w^{max} period; however, they differ in the post-N_w^{max} phase. The curve of NGS show a sharp drop in the post-N_w^{max} phase which corresponds to fast disposal of the weak opinions from the system, in contrast, the $N_d(t)$ curve corresponding to minimal NG shows a slower disposal of the competing opinions. As the removal of opinions from the system corresponds to more successful interactions, the average success rate $S(t)$ in NGS shows a steeper rise post-N_w^{max} period than the $S(t)$ curve for minimal NG due to the dominance of the opinions causing the elimination of the weaker lot quickly from the system.

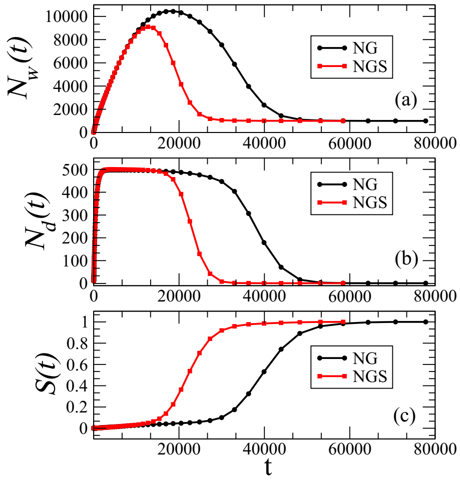

Fig. 3. Comparison of the emergent properties of the system in NG and NGS model. The temporal evolution of (a) the total number of opinions $N_w(t)$, (b) the number of different opinions $N_d(t)$, (c) the average success rate $S(t)$ with $N = 1000$ for NG and NGS model. The NGS model shows faster agreement than NG. All the datapoints on the curve are averaged over 100 simulation runs.

There are two fundamental aspects that depends on the population size N. The first is the time needed by the population to reach the global consensus i.e., the convergence time t_{conv}. The second concerns the cognitive effort in terms of memory required by each agent in achieving this dynamics. This quantity reaches its maximum in correspondence with the peak of the $N_w(t)$ curve. Fig 4 shows a comparative study of the models in terms of the scaling behavior of the global quantities. All the above global quantities N_w^{max}, t_{max} and t_{conv} scales as $N^{1.5}$ in the minimal game while in NGS, the memory peak and the time to reach the peak scale roughly as $N^{1.4}$; t_{conv} on the other hand, scales as $N^{1.3}$ which is faster compared to minimal NG.

B. Scale-free networks

Social networks are far from being fully-connected. Most of them show a large skew in the distribution of node-degrees resulting in the so-called scale-free networks. In this section we will study the dominance effect on the Barabási & Albert

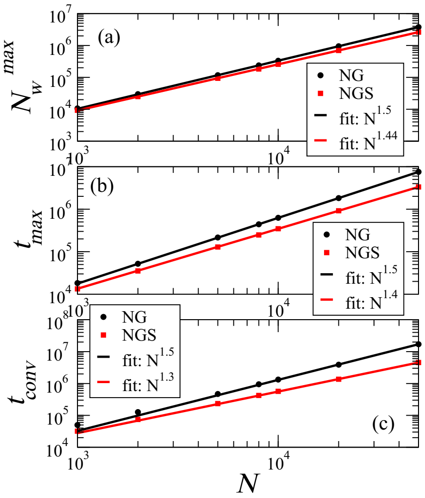

Fig. 4. Comparison of the scaling properties of the system in NG and NGS model. The scaling of (a) the memory peak N_w^{max}, (b) the peak time t_{max} and (c) the convergence time t_{conv} for NG and NGS model are reported here. The model NGS performs better in all aspects.

(BA) network which follows a scale-free degree distribution. Since low degree nodes form a vast majority in such networks, any randomly chosen node is, with high probability, a low degree node. The neighbors of this low degree node, however, should be high degree nodes (since the high degree nodes tend to be connected to almost all low-degree nodes by virtue of their high degree). Hence, in order to reflect the topological dominance of the high degree nodes in the game dynamics, we choose the hearer first and then select the speaker from its neighbor so that the low degree nodes likely become the hearer while the high degree nodes with high probability become the speaker. As inventions take place only at the early stage of the dynamics, choosing hearer first and then the speaker ensures that the dominant people mostly invent opinions and these opinions, in turn, will have higher weights which is a reasonably realistic assumption. However, this variant of pair selection has already been studied and known to influence the dynamics [13]. Incorporating weights on the opinions can be thought of as a second level dominance which is supposed to lead to an even faster agreement. With this modification in the pair selection strategy, we study the dominance effect of NGD and NGS in pursuit of faster agreement.

Fig 5(a) shows the temporal evolution of $N_w(t)$ for NGS and NGD in comparison to the minimal NG. Both the dominance based models indicate a much faster convergence than the minimal case. Another important observation is that N_w^{max} is slightly higher in case of NGD and NGS than the minimal game. This is a consequence of the fact that the low degree individuals who form the majority of the population are chosen as hearers. Most of these hearers encounter failures in the early stage of the dynamics thus increasing $N_w(t)$. In fact, the number of candidate hearers could be potentially much larger than the mean-field case thus increasing the overall value of N_w^{max} in the dominance based models at the early stage of the dynamics where most of the interactions tend to be failures (affecting mostly the hearers). The invention of lower number of opinions shown in fig 5(b) is due to the active role played by the hubs (high degree nodes) in the initial invention process. The hubs are mostly chosen as speaker and hence invents lower number of opinions being minority in the population. Among NGS and NGD, NGD performs better in achieving faster convergence than NGS which is reminiscent of the fact that the weight associated to the opinions in NGD is benefited from the topological structure while opinions in NGS are not. Due to initial heterogeneous dominance, the total number of different opinions are also less in NGD. The $S(t)$ curve (see fig 5(c)) also shows a steeper rise for the case of the dominance models corresponding to faster removal of weak opinions from the system.

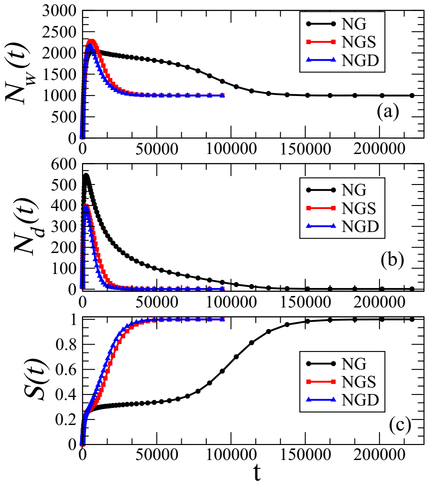

Fig. 5. Comparison of the emergent properties of the system in NG, NGS and NGD models. The temporal evolution of (a) the total number of opinions $N_w(t)$, (b) the number of different opinions $N_d(t)$, (c) the average success rate $S(t)$ with $N = 1000$ and $\langle k \rangle = 8$. The NGD model shows fastest agreement among the three variants. All the datapoints on the curve are averaged over 100 simulation runs for each of 20 network realizations.

In fig 6, the scaling behavior of the global quantities are reported for NG, NGS and NGD models. While all these three models have a roughly same scaling of $O(N)$ for N_w^{max} and t_{max}, they considerably differ in the scaling of t_{conv}. For $\langle k \rangle = 8$, t_{conv} scales as $O(N^{1.35})$ in NG whereas it scales as $O(N^{1.17})$ and $O(N^{1.05})$ for NGS and NGD model respectively (see table I). The scaling laws observed for the convergence time in NG is a robust feature that is not much affected by further topological details, such as the average degree. However, the scaling laws in NGD and NGS are affected by average degree $\langle k \rangle$. In general, the scaling for t_{conv} in NGD is $O(N^{1.1 \pm 0.05})$ while in NGS, it scales as $O(N^{1.2 \pm 0.05})$ for various $\langle k \rangle$ and is considerably better than the minimal NG in all cases.

In fig 7, we try to depict the dependence of t_{conv} on average degree $\langle k \rangle$ in the dominance models as well as in minimal

TABLE I

SCALING RELATION OF THE GLOBAL QUANTITIES IN SCALE-FREE NETWORK.

	N_w^{max}	t_{max}	t_{conv}
NG	N	N	$N^{1.35}$
NGS	N	N	$N^{1.2\pm0.05}$
NGD	N	N	$N^{1.1\pm0.05}$

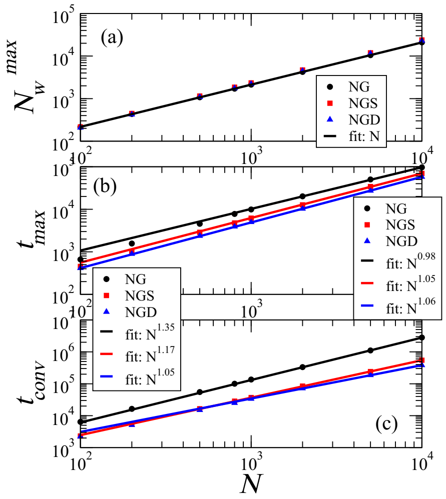

Fig. 6. Comparison of the scaling properties of the system in minimal NG, NGS and NGD model. The scaling of (a) the memory peak N_w^{max}, (b) the peak time t_{max} and (c) the convergence time t_{conv} are reported here. The data points on the curve are averaged over 100 simulations for each of 20 network realizations.

NG. Fig 7(a) shows the dependence of t_{conv} with the average degree for various N in case of the NGD model. With increase in average degree, the memory usage is increased but the system comes to agreement faster. However, if one increases the average degree unboundedly, the scaling of the memory peak and time peak gets affected since the system approaches the mean-field very soon. Similarly, for NGS model (fig 7(b)) also one observes similar degree dependence of t_{conv} for various values of N. Fig 7(c) indicates the case of the minimal NG for comparison which also shows that t_{conv} improves slightly with increase in $\langle k \rangle$ however, always maintaining the scaling of $N^{1.35}$ across different system sizes.

C. Real-world networks

In this section, we focus on real-world networks. With the proliferation of online social networks (OSN), human behavior has profoundly been affected by the influence of individuals in social network via friendship links that constitutes the network. With the growing popularity of these online social networks and with millions of users interacting through posts and tweets, these OSNs are becoming platforms for inter-mixing of social ideologies, traditions and attitudes. Similarly, new opinions are invented almost every second which propagate through friendship interactions on these social networks. The opinions compete with themselves, some die and a very few emerge out as the dominant opinions and compete among themselves for a long time in order to have a global consensus. However, it may not always be possible for all the individuals to come to agreement on a particular issue due to the stubbornness or indifference of individuals towards a particular opinion. Nevertheless, in this study, we assume that all individuals co-operate with each other to eventually arrive at an agreement on an opinion.

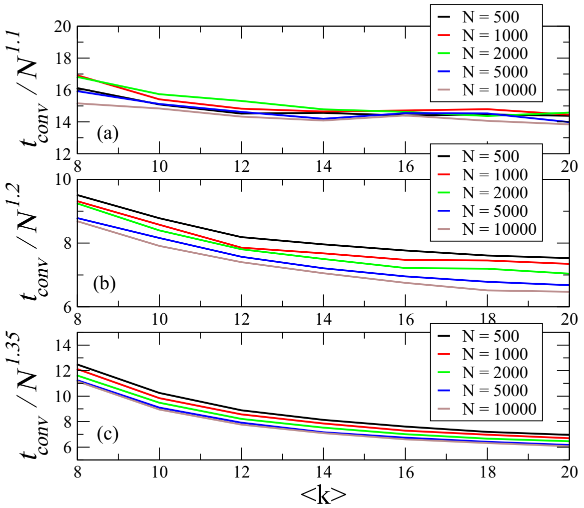

Fig. 7. Degree dependence of convergence time. t_{conv} suitably rescaled by (a) $N^{1.1}$, (b) $N^{1.2}$ and (c) $N^{1.35}$ vs $\langle k \rangle$ in NGD, NGS and NG respectively for various N = 500, 1000, 2000, 5000, 10000 are reported here. The data points on the curve are averaged over 100 simulations for each of 20 network realizations.

We use Facebook snapshot data, as the representative OSN, obtained from Online Social Networks Research group of MPI-SWS, Germany [14]. The dataset was collected between December 29th, 2008 and January 3rd, 2009. There are 63392 nodes and 1633772 friendship links among them after sampling out the largest component from the data. The degree distribution of the facebook dataset shows a power-law behavior (see fig 8(a)).

We perform a similar comparative study of the three models on this real-world network as we did on the synthetic networks. Fig 8 depicts the evolution of the emergent properties of the system with time. Fig 8(b) shows the temporal evolution of N_w where we find a long late-stage plateau in all the models. This long plateau corresponds to a “tug-of-war” situation among the opinions with the total memory size and number of different opinions taking a huge time to stabilize (see fig 8(c)) . In fact, for minimal NG, as we shall see, the system does not stabilize even after 8×10^8 games. This behavior is due to the fact that social networks are inherently modular. These networks typically consist of a number of communities; nodes within communities are more densely connected, while links bridging communities are sparse. The effect of the community structure plays a dominant role in the emergence of the long-lasting multi-opinion state at the late stage of the dynamics which has also been observed by Dall’asta et al. [15] and Lu et al. [7].

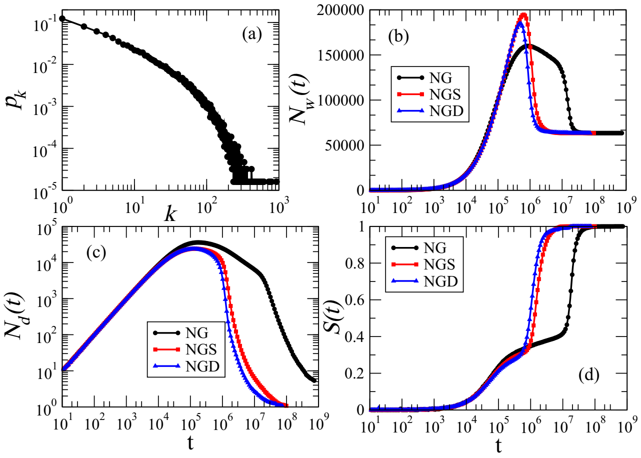

Fig. 8. (a) Degree distribution of Facebook dataset. The temporal evolution of (b) the total number of opinions $N_w(t)$, (c) the number of different opinions $N_d(t)$, (d) the average success rate $S(t)$ on the facebook dataset.

In fact, each community reaches internal consensus fast but the weak connections between communities are not sufficient for opinions to propagate from one community to the other leading to such states which are also referred to as "metastable states" in the language of statistical physics. The behavior of the system on this plateau can be characterized by a series of success events with no change in the inventory. Formally, a metastable state is a state of the dynamics where global shifts are always possible but progressively more unlikely and the response properties depend on the age of the system [16].

As the long plateau is due to the modular topology, all the three models show an existence of it. However, the length of the plateau is significantly reduced in NGD as well as NGS compared to the plateau corresponding to NG. Even after 8×10^8 games, we see that the minimal NG dynamics is trapped into a multi-opinion state ($\sim$ 7 opinions) (see fig 8(c)) with total memory size close to N (see fig 8(b)). This corresponds to the fact that every agent more or less have a single opinion with them but they are non-unique. However, due to dominance of opinions, this phenomenon does not feature so strongly in NGD and NGS resulting in an overall faster agreement (fig 8(c) and fig 8(d)).

IV. CONCLUSIONS AND FUTURE WORKS

In this paper, we studied Naming Game as an opinion formation model on social networks. We have proposed two models NGS and NGD to capture the dominance of opinions in the Naming Game. We show that preferentially choosing dominant opinions invented by dominant individuals in the local interactions can lead to faster agreement compared to the base case where opinions are chosen randomly.

There are quite a few interesting directions that can be explored in the future. One such direction could be to investigate the effect of stubbornness or trust/distrust on opinion adoptation. Another direction could be to study these models on weighted graphs to incorporate preferential connections for opinion propagation. Other direction could be to study the impact of random events on the dynamics.

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